ON THE SECOND-ORDER STEADY MOTIONS OF 3D BODIES IN WAVES

A.Papanikolaou, G.Zaraphonitis
Dept. of Naval Architecture and Marine Engineering
N T U A
Athens 10682, GREECE

SUMMARY

The paper deals with the evaluation of drift motions and loads of arbitrarily shaped 3D bodies freely oscillating in response to regular waves. The employed calculation method is based on a second-order perturbation theory and the solution of resulting BVP's of potential theory in the frequency domain by a pulsating source distribution technique. The required second-order hydrodynamic forces are obtained through an integration of pressure over the intstantaneously wetted hull surface (near field method).

Emphasis has been placed on the derivation of generalized formulas independently of the location of the centers of mass, flotation and coordinate origin [1]. The equations of motion have been set-up to the second-order and solved for the drift motions and loads of a standardized containership (S7-175, 15th ITTC Seak. Comm.) in oblique seas.

The calculated steady deviations and loads are complicated functions of the ship and wave characteristics. They describe various interesting nonlinear characteristics of ship motions, mooring lines or bow thrusters loads and of dynamic stability.

STATEMENT OF THE PROBLEM

The mathematical formulation of the hydrodynamics of a 3D body at zero forward speed, oscillating in response to a regular, possibly steep, wave has been addressed elsewhere [1] and the details can be omitted herein.

Second-Order Near Field Method

Summarizing some results of [1], the "near-field" method by Pinkster and van Oortmerssen [2] has been re-developed and extended to the treatment of bodies with arbitrary position of the mass centroid, the center of flotation and with no restriction as to the choice of the coordinate system. The derived formulas for the drift forces and moments exhibit certain differences to Ref. [2], especially when the aforementioned centroids for mass, flotation and coordinate system are different from each other and not at SWL. Sample calculations for the vertical drift forces on a circular dock (see Fig.1, [1]) show particular effects of the differences in the developed formulas.

Equations of Motion

The equations of motion of the freely oscillating 3D body responding to a regular wave train with finite amplitude oscillations in six-degrees of freedom, can be set-up by applying Newton's Law in the $O x_1 x_2 x_3$ inertial system (see Fig.2):

$$\frac{d}{dt} (M \dot{\vec{x}}_G) = \vec{F} \tag{1}$$

and

$$\frac{d}{dt} (\vec{\Omega}) = \vec{M}_G \tag{2}$$

where M stands for the mass of the body, $\vec{x}_G$ for the position vector of the mass centroid, $\vec{F}$ for the externally applied hydrodynamic force, including gravity. In (2) $\vec{\Omega}$ means the angular momentum of the body and $\vec{M}_G$ the applied external moment with reference to the mass centroid G.

After some manipulations, considering the definition of the modified Euler angles in the order yaw, pitch and roll and using perturbation theory, thus splitting up the motions into first-and second-order contributions, we obtain for the second-order motions:

$$M\ddot{\vec{x}}_G^{(2)} = \vec{F}^{(2)} \tag{3}$$

and

$$I\dot{\vec{\omega}}_2^{(2)} = \vec{M}_G^{(2)} - I\dot{\vec{\omega}}_1^{(2)} - R^{(1)}(I\dot{\vec{\omega}}^{(1)}) - \vec{\omega}^{(1)} \times I\vec{\omega}^{(1)} \tag{4}$$

where $\vec{x}_G^{(2)}$ means the second-order motion of the mass centroid and $\vec{\omega}_2^{(2)}$ stands for the second-order angular velocity in the $O'x_1'x_2'x_3'$ body-fixed system, which is related to $\vec{\omega}^{(2)}$ by:

$$\vec{\omega}^{(2)} = \vec{\omega}_2^{(2)} + \vec{\omega}_1^{(2)} \tag{5}$$

where

$$\vec{\omega}_2^{(2)} = \begin{vmatrix} \dot{\xi}_4^{(2)} \\ \dot{\xi}_5^{(2)} \\ \dot{\xi}_6^{(2)} \end{vmatrix}, \quad \vec{\omega}_1^{(2)} = \begin{vmatrix} -\dot{\xi}_6^{(1)}\xi_5^{(1)} \\ \dot{\xi}_6^{(1)}\xi_4^{(1)} \\ -\dot{\xi}_5^{(1)}\xi_4^{(1)} \end{vmatrix}, \quad \vec{\omega}^{(1)} = \begin{vmatrix} \dot{\xi}_4^{(1)} \\ \dot{\xi}_5^{(1)} \\ \dot{\xi}_6^{(1)} \end{vmatrix} \tag{6}$$

The right-hand-sides of (3) and (4), $\vec{F}^{(2)}$ and $\vec{M}_G^{(2)}$ are given in the appendix A. Also in (4), I means the inertia tensor and $R^{(1)}$ the first-order transformation matrix between the inertial and body system [1].

Taking the time averages of (3) and (4) over one period and imposing zero deviations for the modes lacking internal restoration (surge, sway and yaw) we obtain a coupled system of linear algebraic equations for the second-order steady deviations in heave $\xi_{03}^{(2)}$, roll $\xi_{04}^{(2)}$ and pitch $\xi_{05}^{(2)}$.

DISCUSSION OF RESULTS

In the present paper special attention has been paid to the evaluation of the steady deviations and loads of a standardized ship in oblique, regular seas.

The S7-175 containership model (ITTC Seak.Comm.) has been discretized by a total of 2 x 95 surface elements, using the code described in [3](Fig.3). The angle of incidence and length of the exciting waves have been varied systematically over a wide range and some results thereof are shown in the following figures 4 - 9.

It is of particular interest to study the complicated effects of wave frequency, angle of incidence and eigenfrequencies of the different modes (roll, pitch, heave) on the aforementioned second-order effects. Especially, the calculated steady yaw moment, Fig.9, seems to turn the ship with her side to the waves, for 30° and 60°. But for 90° (beam waves), the steady yaw moment takes opposite sign. Also note the preservation of the positive sign for the longitudinal drift force (Fig.7), independently of the heading and length of the exciting wave, in contrast to [4].

./.

It should be mentioned that similar effects have been detected much earlier by Newman [4], who extended the "far-field" method of Maruo to include the steady yaw moment, besides the horizontal drift forces. However, since of the simplified slender body theory, employed for the solution of the fundamental BVP's, some information about the real body behaviour has been lost, e.g. roll resonance, body asymmetry with respect to midship, etc.

In current work the characteristics of the Series 60, block 0.60, hull, studied before numerically by Newman and experimentally by Spens and Lalangas [5] are recalculated aiming to prove a closer agreement between theory and measurements.

REFERENCES

[1] Papanikolaou, A., Zaraphonitis, G., "On an Improved Near Field Method for the Evaluation of Second-Order Forces Acting on 3D bodies in Waves", Prepr., 4th IMAEM Congress, Varna, 1987.

[2] Pinkster, J.A., van Oortmerssen, G., "Computation of First and Second-Order Wave Forces on Oscillating bodies in Regular Waves", Proc. 2nd Int. Conf. on Num. Ship Hydrod., Berkeley, 1977, pp. 136-159.

[3] Papanikolaou, A., "On the Evaluation of Motions and Loads to Arbitrary Bodies in Waves", Proc. Int. Symp. on Ocean Space Util.'85, Tokyo, 1985, pp. 75-86.

[4] Newman, J.N., "The Drift Force and Moment on Ships in Waves", Journ. Ship Res., Vol. 11 , 1967, pp. 51-60.

[5] Spens, P.G., Lalangas, P.A., "Measurements of the Mean Lateral Force and Yawing Moment on a Series 60 Model in Oblique Regular Waves", Davidson Lab. Rep. 880, 1962.

APPENDIX A

RHS of Equ.(3) (after[1])

$$\vec{F}^{(2)} = \vec{F}_{\Phi}^{(2)} + MR^{(1)}\ddot{\vec{x}}_G^{(1)}$$

$$-\rho g A_{WP}[\xi_3^{(2)} + x_{2F}\xi_4^{(2)} - x_{1F}\xi_5^{(2)}]\vec{i}_3$$

$$+\tfrac{1}{2}\rho g A_{WP}\cdot d_O\cdot(\xi_4^{(1)^2} + \xi_5^{(1)^2})\vec{i}_3 \qquad (A.1.1)$$

where

$$\vec{F}_{\Phi}^{(2)} = \rho\iint_{S_O}\Phi_t^{(2)}\vec{n}\,dS + \tfrac{1}{2}\iint_{S_O}|\vec{\nabla}\Phi^{(1)}|^2\vec{n}\,dS$$

$$+\rho\iint_{S_O}(\vec{x}^{(1)}\vec{\nabla}\Phi_t^{(1)})\vec{n}\,dS - \tfrac{1}{2}\rho g\oint_{C_O}\xi_R^{(1)^2}\vec{n}_{WL}^{*}\,d\ell \qquad (A.1.2)$$

RHS of Equ.(4)

$$\vec{M}_G^{(2)} = -\dot{\vec{x}}_G \times \vec{F}_{\Phi}^{(2)} + R^{(1)}(I\dot{\vec{\omega}}^{(1)}) + \rho g A_{WP}\xi_3^{(2)}\left|\begin{matrix} x_{2G}-x_{2F} \\ -x_{1G}+x_{1F} \\ 0 \end{matrix}\right| +$$

$$+\rho g\xi_5^{(2)}\left|\begin{matrix} L_{12}-A_{WP}x_{1F}x_{2G} \\ \nabla(x_{3G}-x_{3B})-L_{22}+A_{WP}x_{1F}x_{1G} \\ 0 \end{matrix}\right|$$

$$+\rho g\xi_4^{(2)}\left|\begin{matrix} \nabla(x_{3G}-x_{3B})-L_{11}+A_{WP}x_{2F}x_{2G} \\ L_{12}-A_{WP}x_{2F}x_{1G} \\ 0 \end{matrix}\right| \qquad (A.2)$$

$$+\tfrac{1}{2}\rho g A_{WP}d_O\cdot(\xi_4^{(1)^2} + \xi_5^{(1)^2})\left|\begin{matrix} -x_{2G}+x_{2F} \\ x_{1G}-x_{1F} \\ 0 \end{matrix}\right|$$

with $L_{ij} = \iint_{A_{WP}} x_i x_j\,dA$

the moments of inertia of the waterplane area.

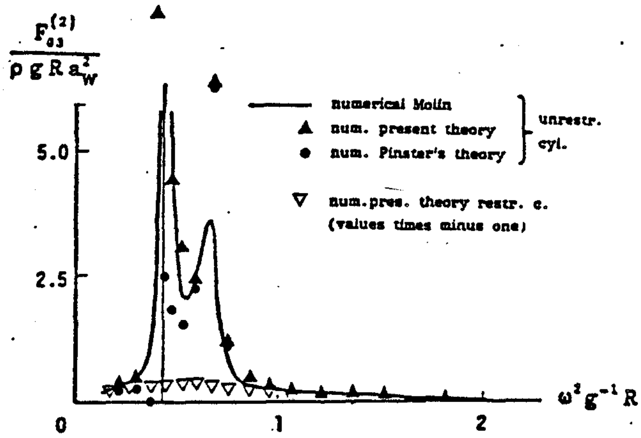

Fig.1. Vertical Drift force on a Circular Dock

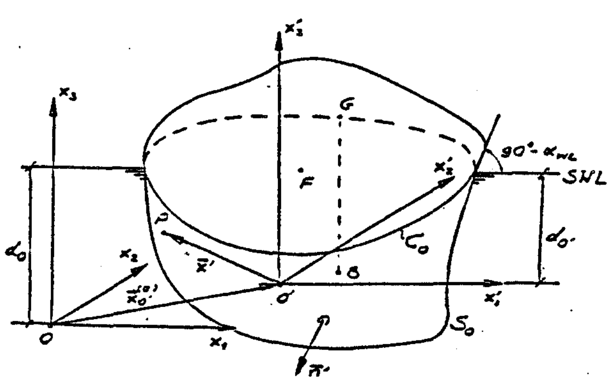

Fig. 2a) Coordinate systems for the body at rest (t=0)

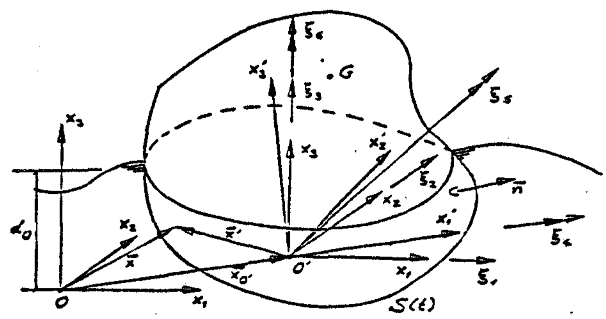

Fig. 2b) Coordinate systems for the body oscillating (t≠0)

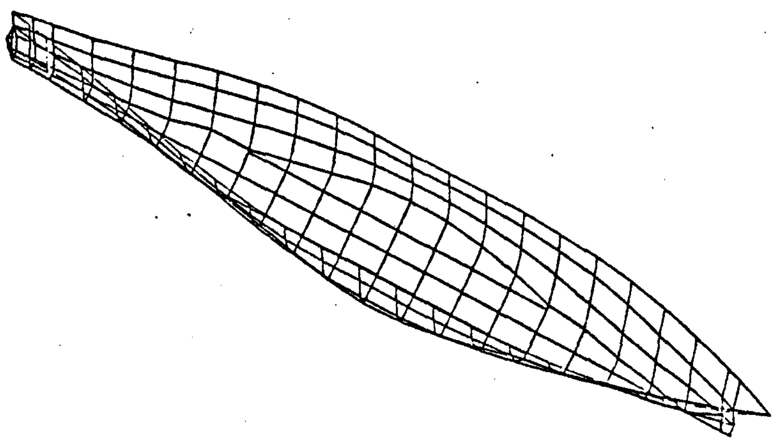

Fig.3. Discretization model of the S175 containership,
N = 2 x 95 surface elements

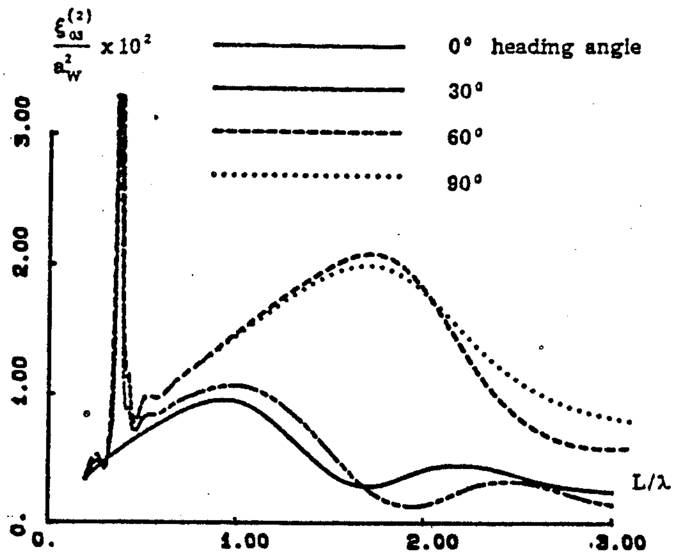

Fig.4. Vertical drift deviation of the S175 model as a function of heading angle β and wave length λ (β = 0 following seas, 90° beam seas)

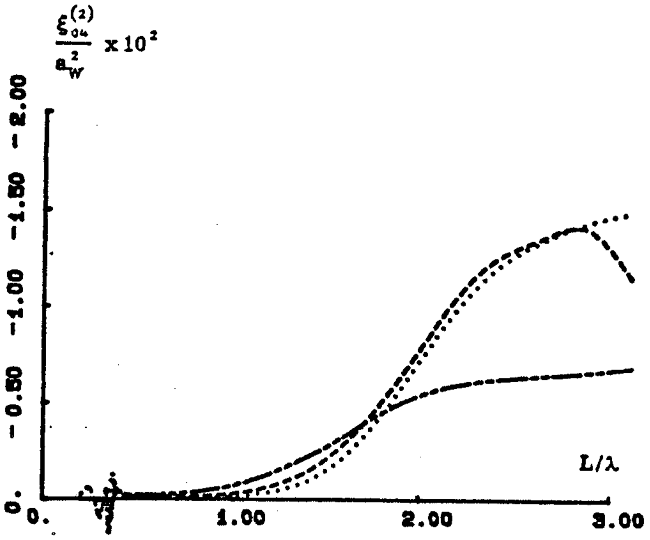

Fig.5. Steady heel angle of the S175 model as a function of heading angle β and wave length λ (β = 0 following seas, 90° beam seas)

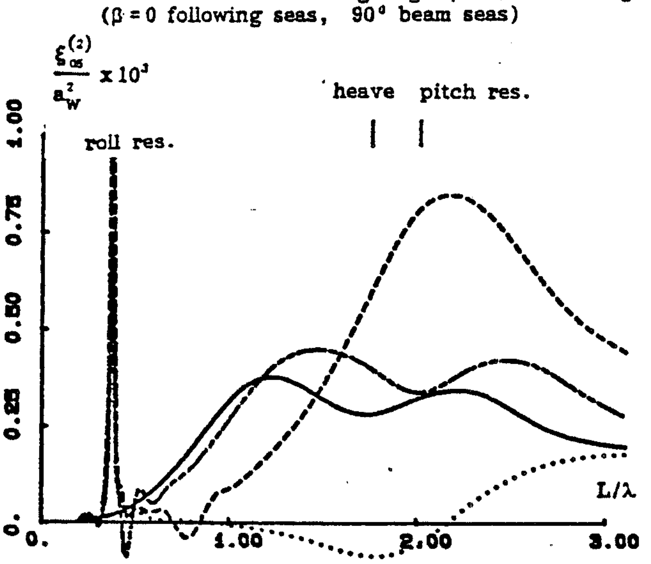

Fig.6. Steady pitch angle of the S175 model as a function of heading angle β and wave length λ (β = 0 following seas, 90° beam seas)

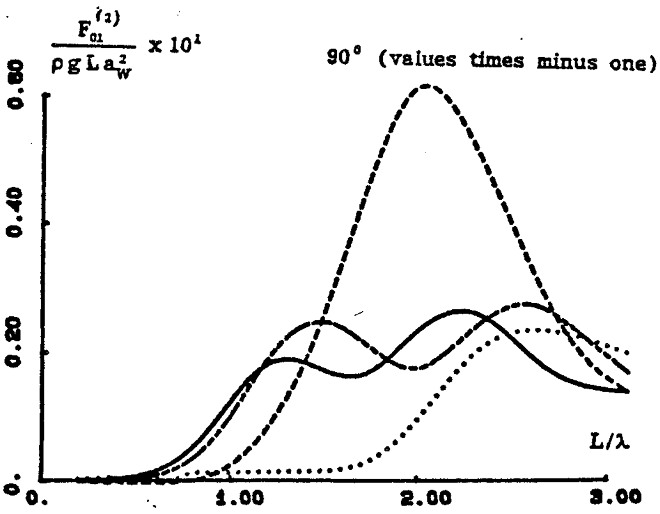

Fig.7. Longitudinal drift force of the S175 model as a function of heading angle β and wave length λ (β = 0 following seas, 90° beam seas)

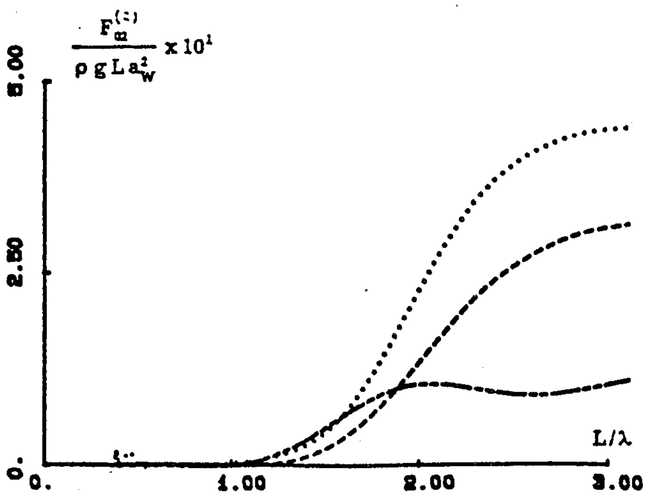

Fig.8. Lateral drift force of the S175 model as a function of heading angle β and wave length λ (β = 0 following seas, 90° beam seas)

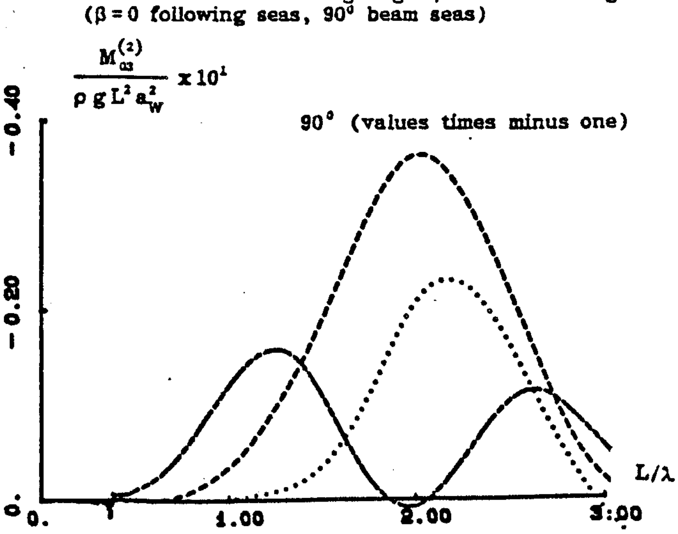

Fig.9. Yaw drift moment of the S175 model as a function of heading angle β and wave length λ (β = 0 following seas, 90° beam seas)

Discussion

Pawlowski: In connection with perturbation schemes of the type presented here, it is perhaps worth mentioning that the overriding principle is to maintain the displacements of the wetted surface uniformly small with respect to the characteristic dimensions of the surface. Therefore for elongated (slender) forms, such as ship hulls, the rigid body displacements cannot be considered all to be of the same order of magnitude. For example for a ship the angular displacements in pitch and yaw modes must be assumed to be as quantities of higher order of smallness than the displacement in roll in order to have linear displacements towards the bow and stern resulting from the pitch and yaw displacements small to the same order as those resulting from roll.