ON THE FIRST- AND SECOND-ORDER TIME-DOMAIN RADIATION PROBLEMS

F. T. Korsmeyer
Department of Ocean Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

In either diffraction or radiation problems we are often more interested in the forces or moments on the body, than the actual potential in the fluid domain. It is in this spirit that Molin (1979) and Lighthill (1979) present formulations for the second-order unsteady force in terms of first-order quantities with no dependence on knowledge of the second-order potential. These formulations, then, do not involve the considerable effort of determining the second-order potential, but they include difficulty of integrating a quantity over the infinite free surface. For instance in Molin's equation the integrand is the product of the inhomogeneity of the second-order free-surface condition and an assisting potential. Recall that it is the device of an assisting potential which allows the recasting of the force formulation in one suitable for the application of Green's theorem.

Similar formulations for both the radiation and diffraction forces can be derived in the time domain. We assume that a general radiation problem has been formulated by expanding the unknown potential in a power series of a small parameter in the usual way. Let us consider in particular, the second-order unsteady force on the body. This force contains a contribution from the pressure due to the second-order potential integrated over the mean position of the body. Since at each order the problem statement for the potential is linear, we can split the second-order potential into two parts: one which satisfies the inhomogeneous free-surface boundary condition and a homogeneous body boundary condition, and one which satisfies complimentary conditions. This latter problem is just the first-order problem revisited with a new right hand side, and therefore it is easily solved. It is the force due to the potential satisfying the inhomogeneous free surface condition which we would like to find without actually determining this potential.

For a body-fixed coordinate system, with x- and y-axes in the plane of the free surface and the z-axis positive up, consider two potentials due to an arbitrary body with motion in mode i. Density, gravity, a body dimension, and body velocity are set equal to 1, and the fluid domain is semi-infinite.

The assisting potential, ψ, satisfies:

$$\nabla^2\psi(\bar{x},t) = 0 \qquad \text{in fluid domain}$$

$$\psi_{tt} + \psi_z = 0 \qquad \text{on free surface, } S_F$$

$$\psi_n = n_i \qquad \text{on body surface, } S_B$$

$$\psi \to 0 \qquad \text{as } \bar{x} \to \infty, \text{ for t finite}$$

$$\psi(x,y,0,0) \text{ and } \psi_t(x,y,0,) = 0$$

and a second-order potential, ϕ, satisfies:

$$\nabla^2\phi(\bar{x},t) = 0 \qquad \text{in fluid domain}$$

$$\phi_{tt} + \phi_z = h(x,y,0,t) \qquad \text{on } S_F$$

$$\phi_n = 0 \qquad \text{on } S_B$$

$$\phi \to 0 \qquad \text{as } \bar{x} \to \infty, \text{ for t finite}$$

$$\phi(x,y,0,0) = 0$$

$$\phi_t(x,y,0,0) = -\frac{1}{2}(\nabla\Phi)^2$$

where

$$h(x,y,0,t) = -\frac{\partial}{\partial t}\left[(\Phi_x)^2 + (\Phi_y)^2 + (\Phi_z)^2\right] + \Phi_t \frac{\partial}{\partial z}(\Phi_{tt} + \Phi_z)$$

in which Φ is the first-order potential for this radiation problem.

Applying Green's theorem to $\phi_\tau(\tau)$ and $\psi(t-\tau)$ gives (omitting spatial arguments):

$$\int_{S_{B,F}} \left[\phi_\tau(\tau)\psi_n(t-\tau) - \psi(t-\tau)\phi_{\tau n}(\tau)\right] dS_{B,F} = 0$$

and invoking the body-boundary conditions and the free surface condition for ψ allows:

$$\int_{S_B} \phi_\tau(\tau) n_i dS_B = \int_{S_F} \left[\phi_\tau(\tau)\psi_{tt}(t-\tau) + \psi(t-\tau)\phi_{\tau n}(\tau)\right] dS_F$$

Integration of this equation in time from zero to t, and using integration by parts to exchange the time derivatives on the right-hand side allows the substitution of h for $\phi_{tt} + \phi_z$ thereby eliminating the second-order potential. Subsequent partial differentiation in time recovers the pressure integral for the force on the left-hand side resulting in:

$$F_i = -\int_0^t d\tau \int_{S_F} \psi_{\tau\tau}(t-\tau)h(\tau)dS_F + \int_{S_F} \phi_t(0)\psi_{tt}(t)dS_F$$

For a body excited harmonically, this formulation recovers the frequency domain result as time goes to infinity, and ψ is essentially the Fourier transform of the frequency domain assisting potential. Like the frequency-domain version, this time-domain method of second order force calculation contains an integral to be evaluated over the infinite free surface. However in this case, for any finite time, the integrand decays rapidly so we can expect accurate results with a truncated range of integration.

The fact that the right hand side is a convolution demands that both large and small time evaluations of the first order potentials be accurate. We assume that for an arbitrary body in any mode of motion that the first order potentials will be found by a boundary element discretization of a Fredholm integral equation. It has been shown by Beck and Liapis (1986) and Korsmeyer (1986) that this technique presents the following difficulty: the manifestation of irregular frequencies in the time domain is an oscillatory error in the impulse response function, L(t), at large time, t. The appearance of this error may be delayed until a later time by a finer discretization of the body geometry; but naturally, for any particular discretization there exists some sufficiently large time where an oscillatory error will be seen. To provide greater accuracy at large t for a reasonable discretization, we can match the mumerical solution to a large t asymptotic solution.

Simon and Hulme (1986) demonstrate that the added mass and damping functions can be expanded to high order for small wave number, k. This is accomplished by a direct expansion of the frequency-domain, radiation-problem integral equation in powers of k and log k, creating a hierarchy of equations all with the "rigid lid" kernel on the left-hand side. This technique can be a useful part of the solution of the linear time-domain problem because the Fourier transform of either the added mass or damping expansions provides the large t asymptotic expansion of L(t). So the oscillatory error can be removed by matching a computed L(t) to this large t expansion. The numerical effort involved is not formidable as all of the damping expansion coefficients may be determined by a single matrix decomposition, although the right-hand sides do become complicated as the power of k increases. Simon and Hulme present the form of the damping expansion; to fourth order in k, it is:

$$B(\omega) = \pi(b_{10}k + b_{20}k^2 + b_{31}k^3\log k + b_{30}k^3 + b_{41}k^4\log k + b_{40}k^4 + \ldots)$$

The Fourier transform of this series can be determined easily from Lighthill (1958) to be:

$$L(t) = 2(2b_{10}t^{-3} - 4!b_{20}t^{-5} - 2(6!)b_{31}t^{-7}\log t + (b_{30}+2\psi(6)b_{31})6!t^{-7}$$
$$+ 2(8!)b_{41}t^{-9}\log t + (b_{40}-2\psi(8)b_{41})8!t^{-9} + \ldots)$$

Where $\psi(n)$ is the digamma function. When $L(t)$ is corrected in this way at large t, the Fourier cosine and sine transforms provide added mass and damping curves free of irregular frequency effects. This makes the time-domain approach not only efficient, but an accurate route to the usual frequency-domain hydrodynamic quantites.

At second order, there are more sources of difficulty than just the behavior at long time. For instance, both first order potentials evaluated on the free surface are rapidly oscillatory very close to the body. But a possible method to improve the accuracy of the second-order force computation may be to correct the first-order potentials at long time. Since the hierarchy of integral equations provides the expansion for the potential, directly, it is possible to construct a large t asymptotic expansion of a time domain potential, again through the application of the Fourier transform.

Results are presented which demonstrate the efficacy of long time asymptotics in both the first and second order problems.

REFERENCES

Molin, B. (1979). Second-order diffraction loads upon three-dimensional bodies. Applied Ocean Research, Vol. 1, No. 4, pp 197-202.

Lighthill, M. J. (1979) Waves and hydrodynamic loading. Proceedings of the Second International Conference on the Behavior of Off-Shore Structures, opening address.

Beck, R. F. and Liapis, S. (1986) Transient motions of floating bodies at zero speed. To be published in Journal of Ship Research.

Korsmeyer, F. T. (1986) On the solution of the radiation problem in the time domain. The First International Workshop on Water Waves and Floating Bodies. pp 61-66.

Simon, M. J. and Hulme, A. (1985) The radiation and scattering of long waterwaves. Symposium on Hydrodynamics of Ocean Wave-Energy Utilization of the International Union of Theoretical and Applied Mechanics. pp 419-432.

Lighthill, M. J. (1958) Fourier Analysis and Generalised Functions. Cambridge University Press, Cambridge.