Time-Domain Analysis of Wave Exciting Forces

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The problem of determining the exciting forces acting on a floating body due to sinusoidal waves has been extensively investigated using frequency-domain analysis. Results for more arbitrary wave systems may then be found using superposition and Fourier analysis. In this paper a technique to solve the exciting force problem directly in the time domain will be discussed.

Solving the radiation problem directly in the time domain has been investigated by many authors. For three-dimensional bodies, Newman(1985) determined the impulse response function for axisymmetric circular cylinders of various radius to depth ratios. Beck and Liapis(1986) and Liapis and Beck(1985) used time-domain analysis and panel methods to solve the radiation problem for arbitary bodies at both zero speed and with forward speed, respectively.

The use of time-domain analysis to solve the exciting force problem has not been widely investigated. Wehausen (1967) developed the analogue to the Haskind relations for a body at zero forward speed in the time domain. Breslin, Savitsky and Tsakonas (1964) and Davis and Zarnick (1964) discussed the concept of an impulsive wave acting on a body. In this paper, a method is presented to solve the exciting force problem directly in the time domain using boundary integral methods.

From linear system theory, it can be shown that the exciting forces acting on a body due to an arbitrary wave amplitude at midship, $\varsigma_0(t)$, has the form of a convolution integral:

$$\begin{aligned} F_j(t) &= \int_{-\infty}^{\infty} d\tau K_j(t-\tau)\varsigma_0(\tau) \\ &= \int_{-\infty}^{\infty} d\tau K_{j0}(t-\tau)\varsigma_0(\tau) + \int_{-\infty}^{\infty} d\tau K_{j7}(t-\tau)\varsigma_0(\tau) \end{aligned} \tag{1}$$

where $K_j(t)$ is the impulse response function for the exciting force in the jth direction. It represents the wave exciting force acting on the vessel due to a unit impulsive wave at midship. As shown, $K_j(t)$ may be decomposed into the Froude-Krylov exciting force and the diffraction exciting force.

Each of these impulse response functions may be determined separately. The Froude-Krylov force is found by an integration of the pressure in the undisturbed wave system over the body surface. To determine the diffraction force an integral equation must be solved. Details of the analysis may be found in King (1987). The following is a discussion of the approach used.

The analysis is restricted to waves coming from forward of abeam. This restriction need not apply in the zero-speed case. However, for the forward-speed case an indeterminacy develops due to the frequency of encounter shift. In following seas there are three possible wavelengths that yield a given frequency of encounter. Since the wave amplitude as seen by the vessel does not uniquely determine the wavelength, the indeterminacy must be resolved with additional information (for example, both wave amplitude and slope), which greatly complicates the analysis.

The impulse response function for the Froude-Krylov exciting force is found by rewriting (1) in terms of an integration of the incident wave pressure over the body surface:

$$\begin{aligned} F_{j0}(t) &= \int_{-\infty}^{\infty} d\tau K_j(\tau)\varsigma_0(t-\tau) \\ &= \int_{-\infty}^{\infty} d\tau\, \varsigma_0(t-\tau) \iint_{S_0} dS\, \hat{p}(P,\tau) n_j \\ &= \iint_{S_0} dS\, n_j \int_{-\infty}^{\infty} d\tau \varsigma_0(t-\tau)\hat{p}(P,\tau) \\ &= \iint_{S_0} dS\, n_j\, p(P,t) \end{aligned} \tag{2}$$

where $p(P,t)$ is the pressure at point P due to the incident wave $\varsigma_0(t)$ and is given by

$$p(P,t) = \int_{-\infty}^{\infty} d\tau \varsigma_0(t-\tau)\hat{p}(P,\tau)$$

The function $\hat{p}(P,\tau)$ is the pressure at P due to an impulsive wave.

For a sinusoidal wave of unit amplitude the velocity potential and free surface elevation in a coordinate system moving with the vessel are given by:

$$\begin{aligned} \phi_0(P,t) &= \frac{ig}{\omega} e^{k(z-i\varpi)} e^{i\omega_e t} \\ \varsigma_0(P,t) &= e^{-ik\varpi} e^{i\omega_e t} \end{aligned} \tag{3}$$

where

$$\begin{aligned} \omega_e &= \omega - kU_0 \cos\beta \\ k &= \frac{\omega^2}{g} \\ \varpi &= x\cos\beta + y\sin\beta \end{aligned}$$

The linearized pressure due to a sinusoidal wave system is given by

$$\begin{aligned} p(P,t) &= -\rho\frac{\partial \phi_0}{\partial t} + \rho U_0 \frac{\partial \phi_0}{\partial x} \\ &= \rho g e^{k(z-i\varpi)} e^{i\omega_e t} \end{aligned} \tag{4}$$

Equating the expressions for the pressure (2) and (4) and using the wave amplitude at the origin yields:

$$\rho g e^{k(z-i\varpi)} = \int_{-\infty}^{\infty} d\tau \hat{p}(Q,\tau) e^{-i\omega_e \tau} \tag{5}$$

Taking the inverse Fourier transform of both sides of (5) gives the desired result:

$$\hat{p}(P,t) = \frac{\rho g}{\pi} \Re \left\{ \int_0^{\infty} d\omega_e e^{k(z-i\varpi)} e^{i\omega_e t} \right\} \tag{6}$$

where $\Re$ denotes that only the real part is to be taken. In taking the inverse Fourier transform, use is made of the fact that $\hat{p}(P,t)$ must be real, which implies that the extension of the left hand side of (5) into the negative encounter frequency range must be complex conjugate symmetric.

The impulse response function is found by integrating $\hat{p}(P,t)$ over the body surface:

$$K_{j0}(t) = \iint_{S_0} dS\, n_j \hat{p}(P,t) \tag{7}$$

It should be noted that $K_{j0}(t)$ does not equal 0 for $t < 0$ but rather it approaches 0 as $t \to -\infty$. However, this does not imply that the system is anticipatory in nature. It results from the fact that when a wave arrives at the origin, it has already contacted the hull of the vessel (and caused a force on the body) at some earlier time. The time delay increases as the wavelength decreases, and hence there is no finite limit.

To determine the impulse response function for the diffracted waves a technique similar to that used for the Froude-Krylov force is used. It is possible to develop Haskind-type reciprocity relations for the time domain (as was done by Wehausen (1967) for zero forward speed), but to our knowledge they have not been developed for the forward-speed case.

Liapis and Beck (1985) developed the integral equation which must be solved to determine the velocity potential on the surface of the vessel due to an impulsive velocity in the jth direction. A similar integral equation can be used to solve the diffraction problem by replacing the body boundary condition with

$$\frac{\partial \phi_7}{\partial n} = -\frac{\partial \phi_0}{\partial n} \tag{8}$$

where ϕ_7 is the diffraction potential. The normal induced velocity due to an arbitrary incident wave system can be represented as the convolution integral:

$$\frac{\partial \phi_0}{\partial n}(P,t) = \underline{n} \cdot \int_{-\infty}^{\infty} d\tau \underline{K}(P,\tau) \varsigma_0(t-\tau) \tag{9}$$

where $\underline{K}(P,t)$ is the vector function for the induced velocity on the body surface as a function of time due to an impulsive wave at midship at $t = 0$. It is found by using the known results for sinusoidal waves and taking the inverse Fourier transform as was done to determine $\hat{p}(P,t)$. The final result is:

$$\underline{K}(P,t) = \frac{1}{\pi} \Re \left\{ \begin{bmatrix} \hat{\imath} \cos\beta \\ \hat{\jmath} \sin\beta \\ \hat{k} i \end{bmatrix} \int_0^{\infty} d\omega_e\, \omega\, e^{k(z-i\varpi)} e^{i\omega_e t} \right\} \tag{10}$$

The impulse response function for the diffraction exciting force may be found by considering an impulsive incident wave such that $\varsigma_0(t) = \delta(t)$. In this case the body boundary condition becomes:

$$\frac{\partial \overline{\phi}_7}{\partial n} = -\underline{n} \cdot \underline{K}(P,t) \tag{11}$$

where $\overline{\phi}_7$ is the perturbation velocity potential due to an impulsive wave at midship at $t = 0$. It is found by solving the governing integral equation using a panel method. Knowing $\overline{\phi}_7$, the impulse response function for the diffraction force is found by integrating the linearized pressure over the body surface. As shown in King (1987), the final result is:

$$K_{7j}(t) = -\rho \frac{\partial}{\partial t} \iint_{S_0} dS\, \overline{\phi}_7 n_j + \rho \iint_{S_0} dS\, \overline{\phi}_7 m_j + \rho \oint_{\Gamma} d\ell \overline{\phi}_7 n_j (\underline{\ell} \times \underline{n}) \cdot \underline{W} \tag{12}$$

where

$\underline{W} =$ fluid velocity vector due to steady translation

$\underline{\ell} =$ unit vector tangential to the waterline curve.

In solving the radiation and diffraction problems, it has been found that nonimpulsive motions give better numerical behavior. The use of nonimpulsive input requires that the linear system theory relations between the input and output be used to determine the impulse response function. System theory can also be used to compare the more usual frequency-domain exciting forces with those predicted from time-domain analysis.

Financial support for this research was provided by the Accelerated Research Initiative of the Office of Naval Research, Contract No. N00014-85-K0118.

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