# BREAKING WAVES SIMULATION

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## ABSTRACT

A numerical model of high amplitude waves is formulated by using the Boundary Integral Method. Emphasis is made on the assumptions to simplify time marching technique. Further extension of the method onto three-dimensional application is possible since only real variables are involved. A two-dimensional spatially periodic breaking wave at finite water depth is given here as an example.

## INTRODUCTION

A numerical technique was developed by Longuet-Higgins and Cokelet in 1976 to simulate a spatially periodic two-dimensional breaking wave at finite water depth. This solution was obtained by solving a boundary integral equation written in terms of complex variables. A mixed Eulerian and Lagrangian form was used to follow the wave's position in space and time to avoid the non-linearity and strong time dependence of the solution due to the free-surface boundary conditions. Later on, Vinje and Brevig (1982) have extended the method to two-dimensional ship motions on large amplitude waves. However, the extension of the method to three-dimensional problems is restricted by the characteristic of the complex variable which is a strictly two-dimensional solution method. The following work develops a solution method that is parallel to the method of Longuet-Higgins but can be extended to three-dimensional applications with necessary modifications.

## MATHEMATICAL FORMULATION

Figure 1 shows the numerical model of a spatially periodic wave at finite water depth. The model is at exactly one wave-length so that the velocity and the potential value on $S_r$ is equal to those on $S_r'$. The bottom of the model can be replaced by a mirror image line so that the number of unknowns can be reduced for a more efficient computation. The combined free-surface boundary condition is given as:

$$\frac{\partial \phi}{\partial n} = - \sin\theta \, u' + \frac{\cos\theta}{g} \cdot \{ \frac{1}{2} \frac{d}{dt} ( | \bar{V} |^2 )' - \frac{d^2 \phi}{dt^2} \} \tag{1}$$

where $\theta$ is the angle of the wave slope, $\frac{d}{dt}$ is the material derivative, and $\bar{V} = (u,v)$.

A finite difference formulation is used to represent the material derivatives in order to march the solutions in time. The material derivative terms in equation (1) are expressed in the following form:

$$\frac{d^2\phi}{dt^2} = \frac{2\phi^0 - 5\phi^{-1} + 4\phi^{-2} - \phi^{-3}}{(\Delta t)^2}$$

where u' and $\frac{d}{dt}\left(|\bar{V}|^2\right)'$ are obtained by a second order extrapolation from previous determined values and the superscript indicates the exact moment where the solution is computed. Substituting equation (2) into (1), the following equation can be obtained:

$$\frac{\partial\phi}{\partial n} = -\frac{2\cos\theta}{g(\Delta t)^2}\phi^0 - \sin\theta\, u' + \frac{\cos\theta}{g}\left\{\frac{1}{2}\frac{d}{dt}\left(|\bar{V}|^2\right) + \frac{5\phi^{-1} - 4\phi^{-2} + \phi^{-3}}{(\Delta t)^2}\right. \qquad (3)$$

The most general form of the Boundary Integral Equation is given as:

$$\phi(p) + \int_{S_r+S_f+S_r'} \phi(Q)\frac{\partial G}{\partial n}\, dS = \int_{S_r+S_f+S_r'} G\,\frac{\partial\phi(Q)}{\partial n}\, dS \qquad (4)$$

where G is the Green's function defined by point P and Q. Point P being the point of interest and point Q being the control point on the boundary. With equation (3) substituted into (4) and the spatially periodicity assumption, equation (4) can be rearranged into the following matrix system:

$$\left[\begin{array}{c|c|c|c|c} I + Gn_{ij} & I + Gn_{ij} + \frac{2G_{ij}}{g(\Delta t)^2} & I + Gn_{ij} & G_{ij} & G_{i\,N+1-j} \\ \hline 1 \;\ddots\; 1 & 0 & 1 \;\ddots\; 1 & 0 & 0 \\ \hline & 0 & & 1 \;\ddots\; 1 & -1 \;\ddots\; -1 \end{array}\right] \left[\begin{array}{c} \phi_j \\ \hline \frac{\partial\phi}{\partial n_j} \\ \hline \frac{\partial\phi}{\partial n_{N+1-j}} \end{array}\right] = \left[\begin{array}{c|c|c} 0 & G_{ij} & 0 \\ \hline & 0 & \end{array}\right] \left[\begin{array}{c} 0 \\ \hline K_j \\ \hline 0 \end{array}\right]$$

and

$$K_j = -\sin\theta\, u' + \frac{\cos\theta}{g}\left\{\frac{1}{2}\frac{d}{dt}\left(u^2 + v^2\right)' + \frac{5\phi^{-1} - 4\phi^{-2} + \phi^{-3}}{(\Delta t)^2}\right\}_j$$

where I is the identity matrix, $G_{ij}$ and $Gn_{ij}$ are the numerical integration of the Green's function and its normal derivative on element j with respect to i, $K_j$ is the sum of the last two terms on the right-hand side of equation (3), and N is the number of elements on $S_r + S_f + S_r'$. Details of the derivation of equation (4) can be found in Calisal, Chan, Rohling (1986).

## RESULTS AND DISCUSSION

Figure 2 presents the deformation of high amplitude waves with various initial wave heights. The water depth to wave length ratio is 0.1 while the time increment is 0.01 sec. 60 elements are used to represent the wave profile. Initial condition is calculated by using linear theory. For wave height to length ratio equal to 0.1, a spilling breaking wave is formed at 1.25 sec. Further increase the wave height to length ratio to 0.125 and 0.15, the waves will deform into the plunging breaking mode at a much shorter time.

From this result, one can see that the use of finite difference representations on the material derivative terms and the extrapolation values for the non-linear terms do provide a convenient numerical technique for the two-dimensional non-linear free surface wave simulations. The assumptions used permit a solution marching in time without using any iterations and corrections. No smoothing technique other than a second order interpolation is used to compute the tangential velocity on the boundary. Numerical stability of the model can be fulfilled provided the value of a non-dimensional number, K, is maintained under 0.50. K is computed by taking the product of the phase velocity and the time step, and dividing by the element length. This non-dimensional number is thought to be associated with the information from travelling a distance larger than the element length per time step.

One final point to be emphasized here is that the presented method consists of real variables only. By including a second angle, $\alpha$, the numerical method can be extended to three-dimensional applications. Although the three-dimensional studies have not been investigated here, no major difficulties are foreseen to limit the modelling of a three-dimensional problem.

## REFERENCES


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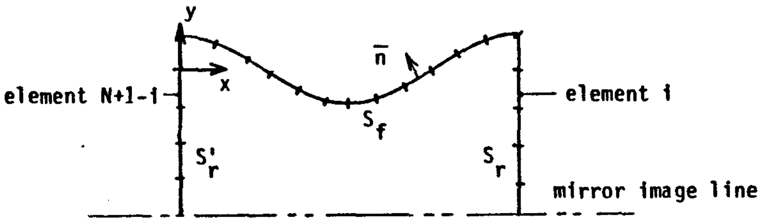


Figure 1 The schematic diagram of the non-linear modelling of a high amplitude wave