Null-field methods for floating cylinders whose cross-sections are elongated

by

P.A. Martin

Department of Mathematics, University of Manchester,
Manchester M13 9PL, England.

Consider a rigid cylinder which is floating horizontally in the free surface of deep water. We shall consider the radiation problem in which the cylinder is forced to oscillate in calm water. We suppose that the oscillations are small and time-harmonic (with frequency ω), and make the usual assumptions of classical hydrodynamics. This leads to a familiar linear boundary-value problem for a velocity potential ϕ. This problem can be treated using many different methods. Here, we shall use the null-field method (Martin 1981, 1984). Specifically, we obtain a new family of null-field methods which are useful for cylinders with elongated cross-sections.

Before describing this work, we begin with a brief description of the standard null-field method. This consists of solving, numerically, an infinite set of moment-like equations for ϕ on the cylinder:

$$\int_{\partial D} \phi \frac{\partial}{\partial n} \Phi_m ds = V_m \quad m = 1, 2, \ldots . \qquad (*)$$

Here, ∂D is the wetted surface of the cylinder, $\partial/\partial n$ denotes normal differentiation, V_m are given numbers, defined by

$$V_m = \int_{\partial D} V\Phi_m ds,$$

V is the prescribed normal velocity on ∂D, and Φ_m are Ursell's multipole potentials: let 0 be the origin of Cartesian coordinates (x,y) so that $y = 0$ corresponds to the mean free surface and y increases with depth; choose 0 inside the cylinder; $\Phi_2(P)$ is the potential at P due to a wave source at 0, Φ_1 is that due to a horizontal wave dipole, and Φ_m $(m > 2)$ are wavefree potentials, e.g.

$$\Phi_{2n+2} = \frac{\cos 2n\theta}{r^{2n}} + \frac{K}{2n-1} \frac{\cos(2n-1)\theta}{r^{2n-1}}, \quad n = 1, 2, \ldots,$$

where $K = \omega^2/g$ and (r,θ) are circular polar coordinates at 0, defined by $x = r\sin\theta$ and $y = r\cos\theta$.

Theoretically, the null-field method is very attractive: unlike simple integral-equation methods (e.g. Frank's 'close-fit' method), the null-field method does not suffer from irregular frequencies. However, simple schemes for solving (*), numerically, are not always successful. Thus, Martin (1981) considered the forced heaving of an elliptical cylinder, but his numerical scheme, using a global basis to represent ϕ on ∂D, did not converge for very thin ellipses. Similiar experiences were reported recently by Takagi et al. (1983), who used different cylinders and a local basis, i.e. they partitioned ∂D into elements and assumed that ϕ was constant over each element. We have used the same local basis for the heaving elliptical cylinder. We found that, numerically, the global bases performed better, i.e. at any given frequency, we could achieve convergence for thinner ellipses.

To obtain a convergent scheme for cylinders with elongated cross-sections, we propose to use a new null-field method, which is obtained by using a different set of multiple potentials: we use Φ_1, Φ_2 and an infinite set of elliptical wavefree potentials, $\tilde{\Phi}_m$, defined by, e.g.

$$\tilde{\Phi}_{2n+2} = 2n\phi_{2n} + \tfrac{1}{2}Kc(\phi_{2n-1} - \phi_{2n+1}), \quad n = 1, 2, \ldots$$

where $m\phi_m = e^{-m\xi}\cos m\eta$, (ξ,η) are elliptic coordinates, defined by $x = c \sinh\xi \sin\eta$, $y = c\cosh\xi\cos\eta$ and c is a parameter at our disposal (the foci are at $x = 0$, $y = \pm c$); we obtain a family of null-field methods by varying c; we could also put the foci at $x = \pm c$, $y = 0$. This set of multipoles was first used by Ursell (1949) for rolling elliptical cylinders; we can prove that it is complete.

Simple numerical schemes for implementing the standard null-field method, (*), are very efficient for cylinders which are 'nearly' circular about 0. We therefore expect that similar schemes for the new methods will be efficient for cylinders that are 'nearly' elliptical. Numerical examples will be given. We note that this approach has also been successfully used for analogous problems in acoustics by Bates and Wall (1977) and, more recently, by Hackman and Todoroff (1985).

References

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Discussion

Kleinman: In acoustics if you go to the analog of wave-free potentials in elliptic or spheroidal coordinates you get Mathieu or spheroidal functions, which are difficult to compute, but here you get remarkably simple functions no more complicated than in the circular or spherical case.

Also in acoustics if you use the regular wave potentials you appear to get better convergence than with the outgoing solutions and it would be interesting to try the calculation with regular waves to see if a similar phenomenon occurs.

Martin: I have not yet used regular potentials to represent ϕ on ∂D, but it seems worth trying.

Newman: Perhaps the regular functions are better for slender bodies because r^n is good near the ends whereas r^{-n} is good in the middle.

Tuck: You are solving a problem which we have known how to solve for more than twenty years with methods which always work. Now you are proposing a method which does not always work; what are we trying to gain here?

Martin: Sometimes the method converges faster than Frank's method. For multi-body problems the method may prove to be more efficient.

Sclavounos: Is it true that the null-field equation method may be more economical in the evaluation of far-field quantities like damping coefficients or the wave field relative to the conventional boundary-integral method? Is this perhaps the reason why it is so popular in acoustics?

Martin: Certainly far-field quantities can be efficiently derived.

X-J Wu: Does this calculation cover all frequencies?

Martin: I have made calculations for a broad range of frequencies, with similar results.

X.-J. Wu Similar ideas but different procedures called interior integral-equation methods for 2D, 3D, and multi-hulled structures have been developed in my previous work ("The interior integral-equation method: I two-dimensional bodies; II three-dimensional bodies; IV multi-hulled bodies", to be submitted). It is my pleasure to display some sample calculations including a 2D rectangle, a 2D triangle and a 3D box (40x40x20 m) which are in very good agreement with the conventional surface singularity-distribution method results. In the 2D cases, irregular frequencies are also observed. However, when using Wu and Price's multiple Green function formulation, there is no irregular frequency in our 2D interior integral-equation method. Comparison with Brown et al's experimental data for a 3D barge, 2.4x0.8x0.105 m, is excellent.

Therefore, when reading about Martin's null-field method (JFM, 1981) I was surprised at the non-convergent problem arising in his approach since there may be no such serious trouble for other existing techniques. Supposing that his derivation is correct, the trouble might be caused by the basic formulations he adopted. That is, the combination of equations

$$\int \phi(Q) \frac{\partial}{\partial n} G(P_-, Q)\, dS = \int V_n(Q)\, G(P_-, Q)\, dS \qquad (1)$$

and

$$G(P_-, Q) = \sum_{m=1}^{\infty} \alpha_m(P_-)\, \Phi_m(Q), \quad for \ r_{P_-} < r_Q \qquad (2)$$

After preliminary study, it has been found that the solution of equation (1) may not be convergent or may not be convergent at the correct value ("On the limitation of the null-field integral-equation method", to be submitted) except that rational treatments are introduced. This conclusion is identical with Martin's sample calculations and Takaygi et al's investigation (1983).

The defect of the singularity-distribution method is the irregular frequency problem. However, in 2D mono, twin or multi-hulled bodies irregular frequencies may be removed by applying Wu and Price's formulation (to be discussed in this Workshop).

The advantage of Martin's null-field method may be the removal of irregular frequencies (although there is no numerical evidence presented), but it brings serious divergent problems. It is well known that when we cannot determine whether or not a proposed theoretical method produces convergent solution, such a technique may be of no practical use. However, I expect that Dr. Martin will improve his method to overcome this defect.

Martin: The infinite system of null-field equations can be properly derived by combining the interior integral equation (Wu's Eq. (1)) and the bilinear expansion of G (Wu's Eq. 2)); it can also be derived by simply applying Green's theorem in the fluid domain to ϕ and Φ_m, for m = 1,2... .

The system of null-field equations and the interior integral equation are equivalent; both are uniquely solvable at all frequencies. To obtain this equivalence it is necessary to satisfy (1) at all points P inside D; satisfying (1) at only a discrete set of points can lead to irregular frequencies (for the corresponding equation in acoustics, see Martin, QJAMAM 33 (1980) 385-396, and references therein).

If I understand Mr. Wu's discussion correctly, he has solved the interior integral equation (1), numerically, for various geometries, and found irregular frequencies. He does not say how he solved (1), but I suspect that there is no conflict with my results. I look forward to reading his four papers on this topic.

Papanikolaou: Referring to Tuck's comments on the applicability of Frank's method to cylinders without vertical entrance at the waterline, in my experience it fails when the flare angle deviates substantially from 90 degrees. We could use the Helmholtz integral-equation method instead of Frank's method.

Hearn: Frank's method can be made to work for a non-vertical intersection at the free surface by careful use of the solid angle concept rather than use of the usual smooth contour assumptions in the integral-equation formulation.

Yue: We have seen a related problem of poor convergence in the context of calculating (free) nonlinear surface waves using a spectral method based on circular harmonic basis functions. Assuming spatial periodicity, the undisturbed free surface can be mapped into a circle. As the instantaneous free surface deviates from this circle, a spectral technique using a Taylor series about the origin results in numerical (and possibly theoretical) difficulties. Your work may throw some light on what we are doing.

Yeung: I am a bit confused about whether or not irregular frequencies are present. Wu showed results with irregular frequencies using the null-field method. I assume there was an error in implementation. I vaguely recall your paper proved that irregular frequencies are absent.

Martin: Irregular frequencies do not occur for this method.