THE RAY-METHOD FOR NONLINEAR SHIP WAVES

by

F.J. Brandsma

(Delft University of Technology, The Netherlands)

The work presented here concerns the use of the ray-method for calculating the wave pattern and the wave resistance of non-thin ships moving at low speed, that is for small values of the Froude number F. Two aspects are treated, Firstly a method for ray-tracing, and secondly the calculation of some excitation coefficients.
The application of the ray-method to ship waves results into a nonlinear equation for the phase function S (see f.i. Keller [3]). The characteristics of this equation are called "rays" and are determined by four ordinary differential equations. For the general case these equations only can be solved numerically. This was done earlier by Yim [2] for bow- and stern waves. The results found here, using a Runge-Kutta method with a self-adjusting step, did not agree with the ones found by Yim for bow waves. No intersections were found of the rays with the waterline. In fig. 1 results are shown for a lens-shaped object. Once the ray-paths are known the phase function itself may be computed, solving an ordinary differential equation along these rays. In fig. 2 some wave-fronts are plotted, for the same object.

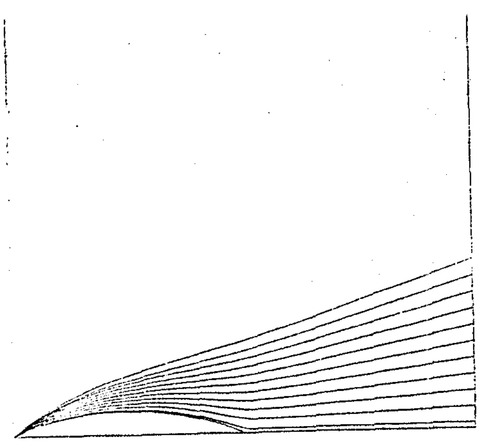

fig. 1

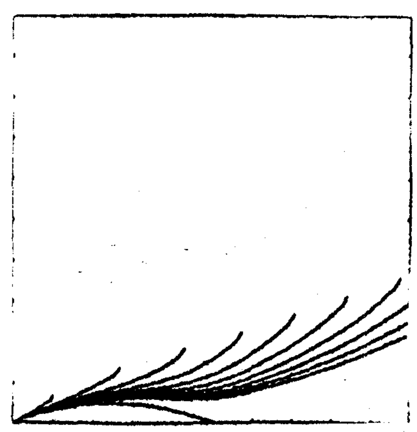

fig. 2

In order to calculate also the amplitude function, excitation coefficients at the source points are needed. For thin ships the Michell theory may be used to obtain these coefficients, as shown by Keller. Here a different approach will be followed for non-thin ships. In [2] Hermans derived an expression for the perturbation potential, combining a perturbation technique with a multiple scale approach. The result was that a source-distribution should be taken over the total free surface. For small values of the Froude number the resulting expression for the wave-height h can be expanded and the leading term is an integral along the waterline of the ship:

$$h(x,z) = \frac{1}{2\pi ik} \int_{-\pi/2}^{\pi/2} \oint_{C_f} A(x,z;\theta,s)\exp[ikS(x,z;\theta,s)]dsd\theta \qquad (1)$$

$$A(x,z;\theta,s) = D(s)(-n_x\cos\theta - n_z\sin\theta)(\phi_\nu(x,z)\cos\theta + \phi_\nu(x,z)\sin\theta)^{-1} \quad (2)$$

$$S(x,z;\theta,s) = \frac{(x-x_0(s))\cos\theta + (z-z_0(s))\sin\theta)}{(\phi_\nu(x,z)\cos\theta + \phi_\nu(x,z)\sin\theta)^2} \quad (3)$$

In which ϕ_ν is the "double-body" potential, $\underline{n}$ the normal vector to the waterline, $k = F^{-2}$ the wave number, and the function D consists of terms containing derivatives of ϕ_ν. It can be shown that this integral when evaluated for $(x,z) \to \infty$, is the same as derived by Baba [1], but it is questionable whether the limiting case is meaningfull because of the use of a multiple scale approach. However for finite distances to the waterline it is assumed that this integral gives a good approximation for the wave height.

After asymptotic expansion of this integral for small Froude numbers (large wave numbers) two types of contributions remain:

1) contributions from bow and stern.
2) contributions from points at the waterline at which stationary phase occurs.

It depends on the geometry of the waterline at bow and stern which of these two types gives the major contribution. For thin ships it can be shown that the major contribution arises from the bow and stern waves, while the order of magnitude in the limiting case will be the same as in the Michell theory.

Excitation coefficients for the use of the ray method may now be derived from this integral by letting (x,z) tend to the waterline. For bow- and stern rays this works well and results will be presented for several geometries.

For the second type of contributions problems arise because of the fact for (x,z) tending to the waterline, the direction of the rays will be tangent to the waterline. For that reason the integral is evaluated first at points at a small distance to the waterline and these points will be used as starting points for the rays.

Conclusions:

The ray method can be used for calculating the wave-patterns of ships moving at low Froude number. The excitation coefficients may be derived by asymptotical expansion of the integral (1). For thin ships the same order of magnitude is found as in the Michell theory.

References:

[1] Baba, E., Wave resistance of ship in low speed, Mitsubishi Technical Bulletin, No. 109, (1976)

[2] Hermans, A.J., The wave pattern of a ship sailing at low speed, Techn. Rep. 84a, Appl. Math. Inst. Univ. of Delaware, (1980)

[3] Keller, J.B., The ray theory of ship waves and the class of streamlined ships, J. of Fluid Mech. 91, pp. 465-488 (1979)

[4] Yim,B., A ray theory for nonlinear ship-waves and the wave resistance, Proc. third int. conf. on num. ship hydr., (1981).

Discussion

Mei: Why do all rays emanate from the bow?

Brandsma: Only the bow waves have been studied so far, but the stern waves and waves emanating from other parts of the waterline are also being investigated.

Tuck: The asymptotic theory for low Froude numbers is clearly a very difficult one, and many outstanding investigators such as Keller and Tulin have tried it without conspicuous success. The present study seems to be along the right lines and could lead to predictions of the correct limiting results as the Froude number tends to zero, especially for quantities other than the wave resistance. Nevertheless, I question the need for low-Froude-number theories. At Froude numbers below about 0.3, the wave resistance is not important compared to viscous resistance. Only when the wave resistance really starts to increase, say for Froude numbers of 0.35, is it comparable to the viscous resistance. But then the more conventional finite-Froude-number theories (even the oft-maligned Michell's integral) do a reasonable job. I am not convinced of the need for a low-Froude-number theory which is bound to be extremely complicated (or else it would have been worked out long ago!) in order to predict something that is in any case negligible.

Wehausen: Froude numbers above .15 certainly are interesting. Tuck is basing his definition of "low Froude number" upon his prior knowledge that wave resistance is insignificant for Froude numbers less than say, 0.10. In fact, in the present context, one cannot say when "small" is small without knowledge of "exact" calculations, which are lacking. A priori, it was quite conceivable that a Froude number of 0.4 would still be small enough. Tuck's point of view is analogous to an earlier one that Michell theory could be automatically disregarded because it applied only to ships as thin as a "knife edge". Neither approximation has turned out to be very practical, but this could not have been known before trying them.